Math 141—Calculus II
Measurable Outcomes

Mathematics Department, UMass Boston

Reference text: Numbers in square brackets refer to sections of *Single Variable Calculus, early transcendentals* by James Stewart, 8th edition.

Note: Outcomes marked **(Optional)** may appear on the final exam with the unanimous consent of all instructors.

1. Area and volume via integration

1(a) Set up the integral representing the area between two curves and evaluate the integral. [6.1]
(Including identifying whether x or y is the convenient choice of the variable in setting up the integral.)

1(b) Be able to write down the formula of a solid using the washer method. [6.2]

1(c) Set up and evaluate the integral representing the volume of a solid of revolution whose axis of symmetry is the x axis using the washer method. [6.2]

1(d) The same for solids of revolution whose axis of symmetry is the y axis or lines parallel to the coordinate axes. [6.2]

1(e) Set up and evaluate the integral representing a solid of revolution whose axis is the y-axis using the cylindrical shell method. [6.3]

1(f) The same for solids of revolution whose axis is the x-axis or lines parallel to the coordinate axes. [6.3]

1(g) Be able to choose between the washer and the shell methods the more convenient one to apply. [6.3]

2. Preparation for the study of integration

2(a) Be familiar with Definition of $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$ and their graphs. [1.5]

2(b) Find the values of other trig functions on such functions. (e.g., $\tan(\sin^{-1} 0.4)$.) [1.5]

2(c) Find the derivatives of functions involving $\sin^{-1} x$ and $\tan^{-1} x$. [3.5]

2(d) Compute integrals whose answers involve $\sin^{-1} x$ and $\tan^{-1} x$. [4.9]

2(e) Evaluate indeterminate forms of type 0/0 using L'Hospital's rule.[4.4]

2(f) Evaluate indeterminate products, differences, and powers by reducing such forms to 0/0. [4.4]

3. Techniques of integration

3(a) Evaluate integrals requiring integration by parts. [7.1]

3(b) Evaluate integrals that require combining integration by parts and substitution. [7.1]

3(c) Express a rational function as quotient plus the remainder over the divisor. [7.4]

3(d) Find the partial fraction decomposition of a proper rational function. [7.4]

3(e) Integrate rational functions of which the divisor factors into (1) a product of distinct linear factors; (2) a product of possibly repeated linear factors.

3(f) Integrate rational functions of which the divisor is a product of linear factors and distinct irreducible quadratic factors, using completing the square when necessary. [7.4]

3(g) Evaluate trigonometric integrals involving powers of $\sin x$ and $\cos x$. [7.2]

3(h) Evaluate trigonometric integrals involving powers of $\tan x$ and $\sec x$ except when the power of $\tan x$ is even and the power of $\sec x$ is odd.

3(i) Know special cases of the last type. (e.g., $\int \sec x dx$.) [7.2]

3(j) Evaluate integrals requiring any of the three cases of trig substitutions. [7.3]

3(k) Combine different techniques of integration to evaluate an integral. [7.5]

4. Improper integrals

4(a) Express an improper integral of type 1 as a limit and evaluate it using L'Hospital if necessary. [7.8]

4(b) Express an improper integral of type 2 as a limit and evaluate it using L'Hospital if necessary. [7.8]

4(c) Determine whether an improper integral is convergent or divergent from its limit. [7.8]

4(d) Understand the convergence and divergence of improper integrals of $1/x^p$. [7.8]

4(e) Determine the convergence or divergence of an imporper integral using the comparison test. [7.8]

5. Sequences

5(a) Find the formula for the terms of a sequence in special cases (e.g., $(-1)^n 3^n 4^{2-n}$). [11.1]

5(b) From a recursive relation to determine terms of a sequence. [11.1]

5(c) Using

- the limits laws
- function replacement and L'Hospital's rule

to determine the limit of a sequence. [1.1]

5(d) Discern a convergent sequence by the monotone convergence theorem. [11.1]

6. Numerical series

6(a) Understand the relation between a sequence and its partial sum sequence. [11.2]

6(b) Recognize a geometric series. [11.2]

6(c) Determine whether a geometric series is convergent or divergent. Evaluate a convergent geometric series. [11.2].

6(d) Evaluate a telescoping series via partial fraction decomposition or occasionally other methods of decomposition. [11.2]

6(e) Recognize a divergent series by the test of divergence. [11.2]

6(f) Apply the integral test to determine whether a series with non-negative terms is convergent or divergent. [11.3]

6(g) Understand the p-series. [11.3]

6(h) Using the corresponding improper integral of type 1 to estimate a series whose terms are non-negative. [11.3]

6(i) Apply the comparison test (usually against a p-series or a geometric series) to determine whether a certain a series whose terms are non-negative is convergent or divergent. [11.4]

6(j) Apply the limit comparison test in place of the comparison test to determine whether a certain series is convergent or divergent. [11.4]

6(k) Recognize a convergent alternating series by the alternating series test [11.5]

6(l) Understand the notions of absolute convergence and conditional convergence for general series. [11.6]

6(m) Determine if certain series are (absolutely) convergent or divergent using the ratio test [11.6]

6(n) Determine if certain series are (absolutely) convergent or divergent using the root test [11.6]

7. Power series and functions represented by power series

7(a) Determine the radius of convergence of a power series. [11.8]

7(b) Determine the interval of convergence of a power series. [11.8]

7(c) Differentiate a power series. [11.9]

7(d) Integrate a power series. [11.9]

7(e) Find the power series representation of functions (centered at 0) reducible to $\dfrac{1}{1-x}$ via algebra. [11.9]

7(f) Find the power series representation of functions (centered at 0) reducible to $\dfrac{1}{1-x}$ via differentiation and integration. [11.9]

7(g) Determine the Taylor series of a function at a point using the definition of a Taylor series. [11.10]

7(h) Determine the MacLaurin series of a function from the definition. [11.10]

7(i) **(Optional)** Use Taylor's inequality to determine whether the Taylor series of the function converges to the function. [11.10]

7(j) Be acquainted with the MacLaurin series of basic transcendental functions such as $e^x, \ln x, \sin x, \cos x$. [11.10]

7(k) Use the MacLaurin series of such functions to find the MacLaurin series of functions that can algebraically constructed from them. [11.10]

7(l) Using Taylor/MacLaurin series of a function to express the integral and the derivative of such a function in terms of power series. [11.10]

8. Differential equations

8(a) Know the definition of an ordinary differential equation and its order and the definition of an initial value problem.[9.1]

8(b) Solve a separable equation. [9.3]

8(c) Solve an initial value problem involving a separable equation. [9.3]

8(d) **(Optional)** Solve a mixing problem. [9.3]

9. Solving more geometric problems using calculus

9(a) Recognize the curve described by parametric equations. [10.1]

9(b) Find the slope of tangent lines to a parametric curve. [10.2]

9(c) Set up the integral representing the arc-length of a parametric curve and evaluate it in special cases. [10.2]

9(d) Set up and evaluate the arc-length of the graph of a function (as a special case of 6(c).) [8.1]

9(e) Find description of points and simple curves in polar coordinates. [10.3]

9(f) Translate between polar and Cartesian coordinates. [10.3]

9(g) Identify the polar curve from a given equation. [10.3]

9(h) Describe a 'fan' region in terms of inequalities involving polar coordinates. [10.4]

9(i) Set up and evaluate the integral representing the area of a 'fan' region in polar coordinates [10.4]

10. (Optional) Elements of complex numbers

10(a) Recognize the real and imaginary parts of a complex number. [Appendix G]

10(b) Find sums, differences and products of complex numbers by algebra. [Appendix G]

10(c) Find the complex conjugate of a complex number. [Appendix G]

10(d) Find the modulus of a complex number. [Appendix]

10(e) Find the real and imaginary parts of the ratio of two complex numbers. [Appendix G]

10(f) Find the geometric interpretation of complex numbers and the algebra on the complex plane. [Appendix G]

10(g) Find the polar representation of a complex number. [Appendix G]

10(h) Use MacLaurin series to understand Euler's formula for complex numbers. [Appendix G]

10(i) Use De Moivre's Theorem/Euler's formula to find powers of a complex number. [Appendix G]

10(j) Use De Moivre's Theorem/Euler's formula to find roots of a complex number. [Appendix G]